DSTL (UNIT-4)

2022-23

Section-A

i. Define normal subgroup.

Section-C

3. Attempt any one part of the following

(a) Let G = (1,-1,i, -i} with the operation of ordinary multiplication on G be an algebric structure, where i = √-1.

(i) Determine whether G is Abelian.

(ii) Determine the order of each element in G.

(iii) Determine whether G is a cyclic group, if G is a cyclic group, then determine the generator/generators of the group G.

(iv) Determine a subgroup of the group G.

(b) Let (G,) and (G',*) be any two groups and let e and e'be their respective identities. If f is a homomorphism of G into G', the prove that (i) f(e) = e' (ii) f(x-1) = [f(x)]-1, XE G

7. Attempt any one part of the following

(a) Prove that the set of residues F = [0, 1, 2, 3, 4} modulo 5 is a field w.r.t. addition and multiplication of residue classes modulo 5. i. e. (F, +5, X5) is a field.

DSTL (UNIT-4)

2021-22

Section - A

c. State and justify "Every cyclic group is an abelian group".

d. State Ring and Field with example.

Section - B

b. Explain Cyclic group. Let H be a subgroup of a finite group G. Justify the statement "the order of H is a divisor of the order of G". I

Section - C

4. Attempt any one part of the following

(a) Define the binary operation on Z by x + y = x + y + 1 for all x, y belongs to set of integers. Verify that (Z,*)is an abelian group? Discuss the properties of abelian group.

(b)(i) Justify that "The intersection of any two subgroup of a group (G,) is again a subgroup of (G,)".

(ii) Justify that "If a, b are the arbitrary elements of a group G then $(ab)^2= a^2b^2$ if and only if G is abelian.

DSTL (UNIT-4)

2020-21

Section - A

c. Define group.

d. Define ring.

Section - B

b. State and prove Lagrange theorem for group.

Section - C

4. Attempt any one part of the following

(a) A subgroup H of a group G is a normal subgroup if and only if $g^{-1}ng$ & H for every heH and ge G.

(b) In a group (G,*)prove that

(i) (a-1)-1 = a

(ii) (ab)-1 = b-1a-1

DSTL (UNIT-4)

2019-20

Section-A

c. Let Z be the group of integers with binary operation defined by a*b=a+b-2 for all a, b ∈ Z. Find the identity element of the group (Z,*)

d. Show that every cyclic group is abelian.

Section-B

b. What do you mean by cosets of a subgroup? Consider the group Z of integers under addition and the subgroup H = {......,-12,-6,0,6 12,...... } considering of multiple of 6

(i) Find the cosets of H in Z

(ii) What is the index of H in Z.

Section-C

4. Attempt any one part of the following

(a) What is Ring? Define elementary properties of Ring with example.

(b) Prove or disprove that intersection of two normal subgroups of a group G is again a normal subgroup of G.